SEEKING A RATIONALE FOR PARTICULAR CLASSROOM TASKS AND ACTIVITY

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This paper traces the evolution of some research projects that relate to the use of a particular type of classroom task as the focus of the learners' mathematical activity. The tasks used are, on one hand, open-ended to allow opportunities for thinking and creativity, and, on the other hand, content specific to ensure that the focus of pupils' activity is not just mathematics generally but the specific content that the classroom program seeks to address. The research grew out of a particular view of learning and teaching, consideration of the motivation of the learners, a perspective on mathematics as the content focus of the learning, and a recognition of constraints operating in classrooms. These perspectives are summarised first, after which there is a discussion of characteristics of the classroom tasks, and some reports of research into their use. Some implications for teacher education are suggested.

Underlying this article is a fundamental assumption is that views on teaching and learning, perspectives on mathematics, and classroom tasks used are directly linked. The following provides a brief elaboration of issues that inform a particular approach to classroom tasks and explains what implications are drawn from each of these for practice.

The article includes:
- a discussion of learning and teaching,
- a summary of some research on motivation of students;
- elaboration of mathematics as the focus of the learning;
- consideration of constraints on classroom processes;
- factors influencing the structure of classroom tasks;
- description of a particular form, termed content specific open-ended tasks;
- a summary of some research into the use of such tasks; and
- some implications for teacher education.

LEARNING AND TEACHING

The tasks described are intended to build on a social constructivist view of learning, to acknowledge views on learning and thinking derived from studies of expertise, to accommodate aspects of how students can be supported in their learning growth, and a recognition that the orientation of the learner is central to the learning.

A social constructivist view, as summarised by Ernest (1994), recognises that knowing is active, "individual and personal, and that it is based on previously constructed knowledge" (p. 2), and that the knowledge is not fixed, rather it is socially negotiated, and is sought and expressed through language.

Ernest listed among the pedagogical implications that there is a need to be sensitive to learners' previous constructions, to seek to identify errors and misconceptions, to foster metacognitive techniques, and to acknowledge social contexts of learners and content. Other implications seem to be that experiences that allow learners to think and create for themselves, and to have opportunity to discuss their interpretations and develop shared meanings will be productive. In other words, the learners' active and social construction of concepts is important.

A view, sometimes seen as competing with this but which is seen here as complementary, was outlined by Bransford, Brown & Cocking (1999) who argued that we should draw on models of learning and thinking from studies of expertise. They suggested that experts have well organised systematic knowledge that is organised to support understanding.

They have fluent access to knowledge that is generally contextualised and embedded in a conceptual framework. A similar perspective was outlined by Owen and Sweller (1988), who explained that experts draw on a large store of knowledge based on their experience. This is interpreted to mean that students need explicit experiences of a range of mathematical concepts, opportunities to link new knowledge to existing conceptual frameworks, and practice at accessing that information. In other words, planned experiences with key mathematical concepts are necessary.

The role of the teacher in each of these is central. As Lerman (1998) said "the metaphor of students as passive recipients of a body of knowledge is terribly limited: so too is the metaphor of students as all-powerful constructors of their own knowledge, and indeed of their own identities" (p. 70). Lerman argued that social relationships associated with classroom learning must also be considered, including roles for informed peers and the teacher. A useful way of thinking about a role for the teacher was proposed by Vygotsky (1978). He described the zone of proximal development to be the "distance between the actual developmental level as determined by independent problem solving and the level of potential development as determined by problem solving under adult guidance or in collaboration with more capable peers" (p. 86). Vygotsky noted that learning is essentially social, and is the process by which children "grow into the intellectual life of those around them" (p. 88). This defines a key role for expert teachers in supporting learners, and identifies an active dimension to those teachers' role.

Lerman argued that the zone of proximal development is within the learning activity rather than separated from it, and that the task is an essential prompt for learners' activity. He suggested that the zone may actually be associated with the classroom, or the group, rather than necessarily the individual learner.

The contentious issue is how this zone can be identified and activated. One metaphor may be a climbing wall, in which holds are colour coded to indicate difficulty. The particular colour indicates where you should climb and is matched to your desired challenge. If you use a hold of any colour the climb is too easy. Perhaps the colour helps you to scale more challenging paths than would be possible without the guide.

Wood (1989) considered the gaps between what is known, and what is recognised, create space for learning: "By means of a number of 'scaffolding functions', one more knowledgeable than the learner may help to bridge such gaps, essentially by activating problem solving in the child" (p. 60). The role of the teacher in identifying blockages, prompts, supports, challenges and pathways is important, but so is the task. The task can help create awareness of potential learnings, and can even be phrased in such a way as to act as hand holds.

The implications for teaching are that with suitable tasks at the appropriate level of challenge, teachers can support pupils in their learning, by providing any necessary support and direction.

MOTIVATION

A unifying characteristic of each of these implications for classroom experience is the centrality of the learners. Directly related to this is the need for learners to take at least some responsibility for their own learning, a key aspect of which seems to be their motivation or disposition to learn.

In a review of research on motivation, Middleton and Spanais (1999) identified five over arching findings. First, it appears that students who are successful are more likely to perceive engagement in mathematics as worthwhile, particularly if the success is attributable to their ability and effort. The perception that lack of success is due to the lack of ability can inhibit students' motivation to learn. This may operate differentially based on gender (e.g., Fennema & Leder, 1990), and culture (Stigler & Stephenson, 1994).

The second result is that motivation towards mathematics is developed early, is learned, is stable, but is amenable to influence by teachers. It seems that the same applies to students' conceptions of mathematics and the way it is learned (McDonough, 1998).

Third, assisting students to develop intrinsic motivation to learning mathematics is superior to providing extrinsic rewards. McComb and Pope (1994) argued that this is particularly important for "hard to reach" students. Mellin-Olsen (1981), in particular, linked motivation to learn with the quality of learning. He described an I-rationale for learning that arises from a view of the school as a way of improving social prospects which he said has a tendency to produce instrumental learning. An S-rationale is based on a view of what is significant knowledge to the student. Relational learning can result if students see some purpose, either in interest, through perceived usefulness, or even as a goal to some other end, for the learning.

This is linked to the fourth of the Middleton and Spanias (1999) findings that inequities exist in the way some groups of students have been taught to view mathematics.

Fifth, achievement motivation in mathematics can be effected through careful instructional design. This relates to the Middleton's earlier (1995) research that identified arousal, personal control, and interest as important in determining the motivation of students to undertake particular activities.

Middleton and Spanais concluded that:

> when individuals engage in tasks in which they are motivated intrinsically they tend to exhibit a number of pedagogically desirable behaviours including increased time on task, persistence in the face of failure, more elaborative processing, the monitoring of comprehension, and selection of more difficult tasks, greater creativity and risk taking, selection of deeper and more efficient performance and learning strategies, and choice of activity in the absence of extrinsic reward. (1999, p. 66)

The implications of this for classroom teaching are that pupils need opportunities for success, that tasks need to present an appropriate level of challenge and difficulty for the students, that increasing students' sense of control, interest, and arousal can help elicit intrinsic motivation, that classroom tasks in which the students choose to participate will be more productive, and that there is a need for consideration of the social context in which the learning takes place.

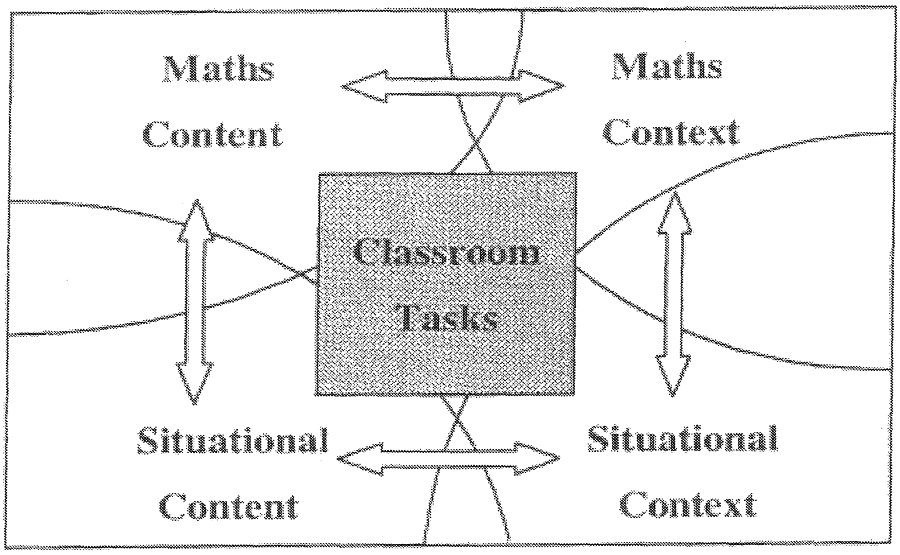
This latter issue is discussed in more detail below, and relates to difficulties experienced by groups of students who are different from the mainstream either socio economically or culturally, and the way that schooling practices are not always compatible with the way such students present and respond.

MATHEMATICS AS THE FOCUS OF THE LEARNING

Another perspective that underlies these classroom tasks is the need to make the mathematics to be learned explicit. There are many tensions associated with this focus. It is widely recognised that situating classroom tasks in contexts within the experience of the students can enhance both the capacity and orientation to engage in working on the tasks (e.g., Australian Education Council, 1991). One tension arises because of the need for students to be able to extract key mathematical elements so that skills and strategies developed can be transferred to other contexts.

The following diagram aims to clarify this notion of the orientation of the tasks. There are four relevant aspects to any problem or task based on a particular situational context, as presented in the following figure:

Figure 1
Relationships between Aspects of Classroom Tasks.



To clarify the intention, the elements are elaborated for the following problem:

> *If each person were to shake hands with each other person in the room once and only once, how many handshakes would take place? (Stacey & Southwell, 1983, p. 38)*

The *mathematical content* of this problem is about recognising that there is a pattern, recognising the n(n-1)/2 relationship, manipulating the symbols, substituting, calculating, and like actions.

The *situational content* is about knowing what is a handshake. This issue is perhaps most relevant to those students with different cultural or linguistic backgrounds, and is sometimes overlooked. Even though this context is probably clear enough, consider how it may be viewed by children who may never shake hands, to women who may shake hands less or greet each other in other ways, to Thai people or to Maoris who may at times use other forms of greeting. This perhaps suggests the possibility of multiple interpretations of particular contexts.

The *situational context* is about people in a room, that you do not shake hands with yourself, and the key variable is the number of people. What we normally think of as real life contexts are included under this heading.

The *mathematical context* is about the concept of trial and error, of strategies such as making the problem simpler, and making a table. Further, a key issue is that we want students to recognise that this context is the same as the mathematical context associated with calculating the number of lines needed to join *n* points on a circle, and of calculating the number of different pairs for table tennis chosen from *n* students. This latter aspect is seen here as the focus for mathematical learning and is often blurred by focus on either the situational context or the mathematical content.

The issue of what is appropriate mathematical content is at the forefront of the "Math Wars" currently raging in the United States. For example, one of the contributors to the debate is a group called *Mathematically Correct.* On their website, a paper by Stevenson (1998) says:

> In our view the NCTM Standards present a vague, somewhat grandiose, readily misinterpreted view of what American children should learn in mathematics. (p. 1)

> In many cases schools elect to emphasise students' general attitudes and fail to provide them with opportunities to learn fundamental mathematics skills. (p. 1)

Likewise, Allen (1998) argued:

> ... mathematics is given structure and coherence by the bones and sinews supplied by definitions, postulates and proof. Make no mistake, a person's problem solving ability depends on how much mathematics he (sic) understands. ... A student who understands the mathematical theory underlying his solution has a master key that will open many locks. (p. 2).

On the one hand, these opinions highlight the need for the curriculum, and classroom tasks, to address specific mathematics content explicitly. On the other hand, just focusing on the mathematical content is too narrow as is elaborated below. In other words, even though the focus may be on particular concepts, the enacting of these concepts requires attention to generalising, problem solving, creating, linking, and synthesising, and these contextual processes are as much a part of the mathematics curriculum as the specific content.

The implications for classroom tasks in this is that there are key ideas in mathematics which should form the focus of the mathematics curriculum and classroom tasks, and the role of the teacher is to frame a program which builds on the mathematical concepts the students have. Situational contexts can help give the students access to the mathematics, but these should not be the end goal of the learning experience nor should they be allowed to blur either the mathematics content or the mathematical context. Providing classroom tasks that allow consideration of the mathematical context is seen as a key element of the teachers' role. Open-ended tasks, because of the possibility of multiple responses and the formation of generalisations, directly promote thinking about the mathematical contexts.

CONSTRAINTS ON CLASSROOM PROCESSES

A further consideration in structuring classroom tasks is related to classroom processes. A helpful framework suggested by the research of Yackel and Cobb (1996), and further elaborated by Herscovics and Schwarz (in press), delineates two complementary norms of activity in mathematics classrooms. *Mathematical norms* refer to the principles, generalisations, processes and products which form the basis of the mathematics curriculum and which serve as the tools of so much other learning. *Socio-cultural norms* refer the usual practices, organisational routines and modes of communication that impact on approaches to learning, types of responses valued, views about legitimacy of knowledge produced, responsibility of individual learners, and acceptance of risk-taking and errors. These socio-cultural norms are too often under-emphasised.

One aspect of the socio-cultural norms is the very real constraints which students naturally impose on the mathematics teacher. It appears that the reaction of students is foremost in the thinking of teachers in planning, during teaching, and in post lesson reviews. Teachers seek to avoid negative student reactions (Shroyer, 1982), to adhere to stated plans (Peterson & Clark, 1978), to minimize risks (Marland, 1986), and to adjust their teaching to the preferred learning style of the students, who exert indirect but real influence over the teacher (Larson, 1983).

Doyle (1986) noted that this influence of students on teachers tends to be negative. He reported that pupils tended to misbehave during tasks that involved higher-order processes such as understanding, reasoning, and problem formulation because they could not anticipate the response expected by the teacher, and the risk of failure was high. In contrast, students worked efficiently on tasks involving recall of algorithms. Doyle argued that, on the one hand, students tried to reduce their risk of failure by seeking to increase the explicitness of task requirements and to reduce the level of accountability, thereby narrowing the demands of a task. Teachers, on the other hand, tended to react by selecting tasks that were familiar and easy.

Desforges and Cockburn (1987) reached similar conclusions. They examined the influence of classrooms processes on mathematics teaching in particular. Among their findings were that students were not interested in each other's opinions and quickly became fidgety during class discussions. Classroom order was seen as a negotiated contract, and teachers were understandably reluctant to threaten the cooperation of students. Children liked the worksheets, which reinforced the teachers' approach. The more the content moved toward unfamiliar work, the more difficult teaching became. Any classroom teaching strategy should seek to address this directly.

Included among the implications for classroom processes are that reducing the censure commonly associated with failure, and therefore the liability associated with risk, is a key. This can perhaps be achieved by explicit attention to classroom sociocultural processes, along with consideration of tasks for which the risk is reduced by, for example, the allowing the possibility of multiple correct responses. It may also be helpful for teachers to emphasise the way that discussion contributes to learning for the students, and the way in which they can benefit from hearing the perspectives of their peers.

A different but highly relevant aspect of these socio-cultural processes is related to the biases inherent the classroom, and the cultural biases inherent in so called progressive teaching methodologies. As Lerman (1998) commented, classroom discourses both "distribute powerlessness and powerfulness" (p. 76). Zevenbergen (1998) referred to the literature outside mathematics education that has recognised the distinct differences in patterns of language use of socially-disadvantaged students and that of the formal school, but noted that only minimal work on this has been carried out in the field of mathematical learning. Zevenbergen argued that procedural processes can contribute to student disadvantage, and noted that pedagogy often renders invisible cultural norms through which meaning is conveyed. Students from backgrounds where there are discontinuities between linguistic registers and societal aspirations of home and school have to decode aspects of classroom processes. For children whose cultural norms are similar to those embedded within mainstream pedagogical practice, the mathematics is usually more accessible while the converse is true for those students whose culture does not fit the dominant classroom routines. Zevenbergen proposed that one solution that appears to enable these students to gain access to the lesson content is to make socio-cultural norms of pedagogy explicit to teachers and, through them, also to children.

While the full implications for this are not discussed here, it is clear that tasks that allow teachers to discuss with pupils the goals and purpose of their activity, that can be accessed in a variety of ways, and that allow for a wide range of legitimate classroom activity, will facilitate an environment in which the teacher can address these other aspects of socio-cultural processes.

STRUCTURING CLASSROOM TASKS

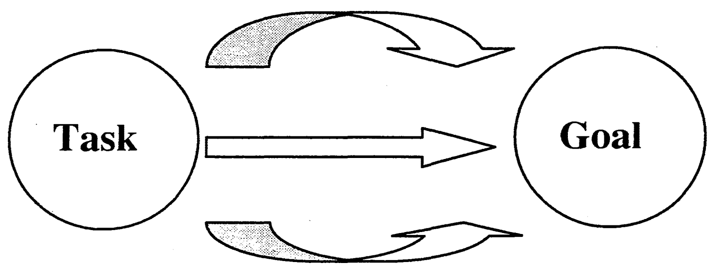
The key argument here is that the above considerations cannot be addressed by emphasising productive classroom strategies such as discussion and group work if the tasks set are restricted to those that only focus on mathematical content or that overemphasise situational contexts. At least some tasks must also address mathematical contexts, as described above, as well as addressing the orientation of the learner and providing opportunities for teachers to address the socio-cultural norms of classroom activity.

Christiansen and Walther (1986) also argued that the tasks posed are critical. They explained that the task set, and the associated activity, serve as the interaction between teacher and learner. They argued that non-routine tasks, because of the interplay between different aspects of learning, provide optimal conditions for cognitive development in which new knowledge is constructed relationally and items of earlier knowledge are recognised and evaluated. It follows that the best tasks are those that provide appropriate contexts and complexity, are open so that they may stimulate construction of cognitive networks, thinking, creativity and reflection, and address significant mathematical topics explicitly.

Sullivan, Warren and White (in press) developed a characterisation of tasks based on the task, the activity, and the goal. They defined the *task* as the statement presented to students that serves as the prompt for their work. Their *activity* is the thoughts and actions in which they engage in response to the prompt. The *goal* is the results the students seek as a product of their activity in response to the task statement. Each has the potential to be open or closed. *Closed* implies there is only one acceptable pathway, response, approach, or line of reasoning. *Open* refers to the existence of more than one (preferably many more than one) possible pathways, responses, approaches or lines of reasoning.

It is assumed here that at least some classroom tasks should stimulate, or at least allow, open activity — that is, students will ordinarily follow different approaches to the task goal. Open activity is seen as fostering some of the more important aspects of learning mathematics, specifically investigating, creating, problematising, communicating, and thinking, as distinct from merely recalling procedures. The following is a representation of this process.

Figure 2
Link between task, goal and activity



This assumption on the openness of open *activity* underlies most literature on problem solving and investigations (see, for example, Anderson, 1996). There is, though, considerable debate about the significance of the openness of the *task* and/or the *goal* (see, for example, Pehkonen, 1997; Wiliam, 1998)

Tasks that have open statements contribute to a broad and rich curriculum and have potential to make a significant contribution to mathematical learning. Nevertheless the tasks discussed here have a focus on specific aspects of the mathematics curriculum. This content specific nature of the tasks implies that the statements need to be closed. The openness in the tasks discussed here arise from the openness of the goals, and the term *open-ended* is used to describe tasks that have such goals.

In other words, the tasks have closed statements, promote open activity, and have open goals or ends.

It should be noted that even though the task statement might be closed, this does not mean that all students must start working on the task in the same place or in the same way.

Indeed the very openness of the goals allows for students of different understanding and backgrounds to approach the tasks in different ways. This point is elaborated further below.

CONTENT SPECIFIC OPEN-ENDED MATHEMATICAL TASKS

It is argued that in order to address the various perspectives on learning, learners, mathematics, and classroom processes, the choice of the task is critical. It is suggested that one option is to consider content specific open-ended tasks.

The term *content specific open-ended tasks* can be illustrated by means of some examples:

> *A number has been rounded off to 5.6. What might be the number?*
>
> *Draw as many triangles (on square paper) as you can with an area of six square units.*
>
> *The mean height of four people in this room is 155cm. You are one of those people. Who are the other three?*
>
> *A ladder reaches 10 metres up a wall. How long might be the ladder, and what angle might it make with the wall?*
>
> *What has the same mass as ten $2 coins?*
>
> *A train takes 1 minute to go past a signal. How long might be the train and how fast might it be travelling?*
>
> *What are some functions that have a turning point at (1,2)?*
>
> *Find two objects with the same mass but different volume.*
>
> *At what times do the hands of a clock make an angle of 90°?*
>
> (For a wide range of such tasks at the primary level, see Sullivan & Lilburn, 1997.)

To explore ways in which such tasks might contribute to a classroom program, it may be helpful to contrast corresponding closed and open-ended tasks. An example of a conventional closed item might be:

> *A rectangle is 10 m long and 5 m wide. What is the perimeter and area?*
>
> A comparable open-ended task is:
>
> *If the perimeter of a rectangle is 30 m, what might be the area?*

This latter open-ended task is different from the conventional perimeter and area question in two major ways. First, it requires a higher level of thinking and engagement than does the conventional question. Traditionally mathematical questions require students to repeat a procedure or recall an algorithm. The open-ended task has the potential to engage students in constructive thinking by requiring them to contrast the related concepts of perimeter and area and to think about relationships for themselves. Another advantage of the open-ended task over the conventional item is that the need for thinking by individual students is made clear to them. The students cannot rely on remembering a rule or simply manipulating formulas. They must think about the concepts, their meaning and the links between them.

Second, the question has more than one possible appropriate answer. Some students might give a single response, others might produce many appropriate answers, and there may be some who will make general statements. The following are some examples of the different levels at which students might respond to the open-ended question:

Type of response	**Sample response**
Single	$56m^2$
Many	$14m^2$, $26m^2$, $36m^2$, $44m^2$, $50m^2$, $54m^2$, $56m^2$ and even $40.25m^2$, $56.25m^2$, etc.
General	Area is $l \times w$ where $l + w = 15$

The openness of such questions offers significant benefits to classroom teachers because of their potential for students at different stages of development to respond at their own level. In particular, the openness of the goals allows easier entry to the task, and also allows for ready extension using similar task statements.

It is proposed that among other advantages for classroom teachers are that open-ended tasks are suited to group learning, and they focus the attention of students onto aspects of mathematics such as generalising, and identifying patterns and relationships. This can include introducing new concepts such as fractional measurements used above. Such tasks allow students to be creative, to work with others in responding to set tasks, and to recognise that problems can have multiple solutions. An important feature of the tasks is that learning occurs as an outcome of the students' explorations and thinking, rather than as a result of listening to the teacher (Sullivan & Clarke, 1991). A further characteristic is that such tasks can be created by teachers for any level, for any topic.

It is interesting to note that this very task was criticised by Wu (1994). Wu's views are liberally quoted on the *Mathematically Correct* website (see, for example, Becker & Jacobs, 1998). Wu claimed that the students would be unable to find complete solutions and therefore the task should not be used in lower levels. This seems to be a somewhat restricted and counter-productive view. There is an implication that students should not work on problems unless they are able to produce one complete solution. It ignores the critical fact that students can learn about mathematics from their explorations of situations. Further, a key component of such an approach to teaching is that, in the review of students' work, other students have the opportunity to hear a range of approaches and responses, and so learn that there are multiple answers and scope for further thinking.

Stevenson (1998) also specifically criticised an example of such a content specific open-ended task. The example that concerned him, when paraphrased into Australian language, would read:

I have some coins in my pocket. I put three of the coins in my hand. How much money might I have in my hand?

Stevenson claimed that such problems are "likely to be of little help to the majority of teachers, teaching the majority of our children" (p. 1).

While this particular open-ended task does not necessarily require a higher level of thinking and engagement than its closed alternatives, it still allows the possibility of practice at addition. It shares the advantage of the area task in that it allows multiple answers and ready extension, as well as the potential for use of pattern. I identified 32 different solution totals. The context could also lead on to investigations like:

> *Assuming coins have the same chance of being drawn, what is the expected amount you would have in three coins?*
>
> *Assuming coins have the same chance of being drawn, how could you model this problem?*
>
> *Making no assumptions about the coins, how could you model this problem.*

In summary, it is argued that content specific open-ended tasks have the potential to provide students with opportunities for the very experiences that seem likely to contribute to a rich classroom curriculum.

SOME RESEARCH INTO THE USE OF CONTENT SPECIFIC OPEN-ENDED TASKS

Some research projects have sought to explore the way that students respond to open-ended tasks, and the way that such tasks can be used in classrooms.

Sullivan, Clarke and Wallbridge (1991) asked pupils at a range of grade levels a variety of open-ended tasks. The tasks were posed in various contexts, with some reponding individually, and others working in groups. Sullivan, Clarke and Wallbridge found that:

- pupils can respond to such tasks at a range of levels especially if prompted by the wording of the task;
- the quality of response within a single grade level varied, and student responses improved in higher grades;
- students at all levels gave better answers to questions which drew on content that had been learnt some years previously;
- students can learn from the activity of working on such tasks;
- students often gave only one response even if they could have given more; and
- the degree of difficulty of open-ended tasks varied markedly from similar closed tasks, with open-ended tasks often being more difficult.

They proposed that programs that used predominantly open-ended tasks may need to be supplemented with specific closed skill practice, and that teacher reviews after open-ended activity may need to be explicit for students generally to gain the benefit of their explorations.

Clarke, Sullivan and Spandel (1992) compared responses of students from a range of secondary levels to open-ended tasks drawn from four discipline areas. They found that:

- the inclination to give single responses (or the reluctance to give multiple responses) is a product of schooling, and not peculiar to mathematics;
- the explicit request of multiple responses produces a significant increase in the response level in all academic contexts;
- the capability to give multiple responses increases significantly with year level, except in the case of English;
- that gender-related differences in response level are evident, and where these exist they favour girls; and
- students in both year 7 and 10 possessed a comparable reluctance to provide multiple responses; but when multiple responses were explicitly requested, there was a significant increase in the proportion of multiple responses offered by all pupils in all four academic domains.

Clarke, Sullivan and Spandel recommended that inferences concerning student learning drawn from student performance on open-ended tasks should take into account students' beliefs concerning appropriate answers in academic contexts or risk mistaking student inclination for student capability.

Sullivan, Bourke and Scott (1997) conducted a detailed investigation of a classroom implementation of a program based solely on content specific open-ended tasks. The program was planned with the teacher to ensure that the tasks were suitable for the class, and was delivered in a way that was compatible with the intentions of the study. Most tasks posed were open-ended, and there were few teacher explanations. Generally the students engaged in personal constructive mathematical activity and there were few management or organisational difficulties created by the approach. Observation of individual students and interviews confirmed these impressions and indicated that teaching based on open-ended tasks is suitable both for students who are confident at mathematics and for those who lack confidence. Students showed significant improvement on a test of closed tasks based on the content of the program and the improvement was maintained after the program. Students also showed an overall improvement on the open-ended tasks although the tendency to give multiple correct responses was not maintained after the program.

Sullivan, Warren and White (in press), using methods based on the work of Hay (in press), compared the responses of large numbers of Year 8 students at comparable closed and open-ended tasks. They found that students responded well to some tasks but poorly on others. In most cases the responses were comparable with results on similar tasks previously published.

One of their goals was to explore the effect of situational context on the students' responses. For the closed tasks, the context seemed to help for one task, but made no difference in tasks that included diagrams. It is possible that the diagrams performed a similar role to the context for closed tasks. For the open-ended tasks, the context helped for two tasks, but seemed to make a third task more difficult.

Another of the goals of Sullivan, Warren and White was to compare responses to closed and open-ended tasks. They used a specific prompt to seek multiple responses. In two task types, students found the open-ended tasks more difficult suggesting that these required thinking above and beyond that required for the corresponding closed tasks. In the other case the open-ended task was easier. An analysis of responses and a breakdown of elements of the tasks seemed to explain these differences. The open-ended tasks that were more difficult required students to link two concepts and to use such links to conjecture and generalise. Such open-ended tasks may serve the role of stimulating students' thinking to higher levels.

Both the context and the open-endedness seemed to affect the focus and student response to the tasks. Sullivan, Warren and White suggested that each of the type of tasks could contribute productively to classroom programs. In some cases the open-ended tasks may serve as a useful preliminary exploration of topics; in other cases, they may be better left until later.

Stephens and Sullivan (1997) reported on results from a project that sought to examine a range of aspects of performance assessment in mathematics (see also Beesey, Clarke, Clarke, Stephens & Sullivan, 1998). Open-ended tasks were used as one component of a number of their performance assessments. They reported that teachers were able to apply the scoring rubric to the tasks, that the scoring allowed the tasks and student performance to be evaluated, and that independent judgments could be made validly.

Sullivan, Warren and White (in press) also argued that both closed and open-ended tasks could be used productively in assessment of student learning in that they require different responses from conventional closed questions and reveal different and important information. Where students give multiple responses, this tells the teacher much about the thinking of the students. Where students give some correct and some incorrect responses this reveals information about the quality of understanding of the students. Because the open-ended tasks reveal different information from the closed tasks, they have potential to enrich assessments.

SUMMARY OF IMPLICATIONS FOR TEACHING

It has been argued that the tasks teachers pose as the basis of classroom activity are directly linked to the type of thinking and engagement of learners. A range of considerations for the design of tasks has been described, and it is proposed that content specific open-ended tasks have potential to address many of these background considerations.

The following summarises the teaching implications for classroom tasks indicated above. Classroom tasks should ideally allow teachers to:

- be sensitive to learners' previous constructions;
- identify errors and misconceptions;

- foster self awareness of learning processes;
- acknowledge social contexts;
- allow opportunities for learners to think and create;
- provide source material for discussion;
- exercise leadership of the learning and provide expert advice;
- allow practice at accessing knowledge;
- foster linking new knowledge to previous conceptual frameworks;
- encourage learners to take responsibility for their learning;
- ensure opportunities for success;
- include an appropriate level of challenge;
- increase students' sense of control and choice, interest, and arousal;
- explicitly focus on specific mathematics concepts;
- emphasise key ideas of mathematics in focusing the curriculum;
- attend to problem solving;
- establish a classroom ethos where students can take risks;
- change the nature of right/wrong as the measure of success;
- create opportunity to discuss students' goals and purpose of activity;
- allow a wide range of legitimate activity;
- consider pupils' perspectives of learning and learning environments; and
- collect rich assessment information.

As has been discussed above, content specific open-ended tasks have potential to address most of these ideals explicitly, and certainly allow the teacher to use the tasks as a springboard.

IMPLICATIONS FOR TEACHER EDUCATION

It is relevant to consider the implications for teacher education. This article has outlined an example of how results from research can be used to inform particular approaches to teaching, provide the tools for evaluating effectiveness of classroom strategies, clarify goals and purpose of particular strategies, and facilitate the study of practice. Since content specific open-ended tasks are simple to create, the approach allows prospective and current teachers significant control over the tasks they use in their teaching.

A further issue is such tasks provide a vehicle for considering the management of the complexity of teaching. Mousley and Sullivan (1997) argued that teacher educators should introduce prospective teachers to a range of dilemmas they will face in classrooms, and highlight both awareness of the tensions and the need to seek reconciliation of tensions to allow action. Open-ended tasks provide suitable source material for this.

Mousley and Sullivan proposed a framework in which prospective teachers would consider that choices are not so much about which poles of dilemmas to emphasise, or even about determining which poles are compatible with conventional wisdom, but are about how to use the dilemmas themselves to strengthen both curriculum and pedagogy. Mousley and Sullivan suggested that it is better that teachers become conscious of such choices, and be able to use alternative approaches to advantage. Rather than seeing different processes as dichotomous, teachers can become aware of the continua associated with each dilemma and move along these strategically, according to situational factors operating at particular times.

Mousley, Sullivan and Mousley (1997) used consideration of dilemmas as a central principle in their use of technology to provide case study material for teacher education students. They created a CD based resource that uses an open-ended task to raise problematic teaching situations and enhance awareness that there are dilemmas to be confronted. The basic proposition which underlies the resource is that the study of particular exemplars of quality practice can stimulate reflection on key components of teaching. It is expected that groups of users will investigate and discuss focus questions that aim to draw out varied pedagogical beliefs and to stimulate sensitive responses.

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Endnote

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